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ТЕЛЕПОРТАЦИЯ И ЛОКАЛЬНАЯ ДЕЙСТВИТЕЛЬНОСТЬ

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Рассматривается процедура квантовой телепортации в рамках алгебраического подхода. Оказывается, что использование гипотезы нелокальности квантового измерения не обязательно для описания этой процедуры. Изучается вопрос о том, какие материальные объекты являются носителями информации для квантовой телепортации. Статья рекомендована к печати программным комитетом международной научной конференции “Математическое моделирование и вычислительная физика 2009” (MMCP2009, http://mmcp2009.jinr.ru).

Ключевые слова: телепортация, алгебраический подход.

A specific method for transferring information is called “teleportation” (see, e.g., [1]). This method has some mysterious features even in the purely scientific literature, because the material information carrier is not clearly indicated. Instead, nonlocality, which is supposedly inherent in quantum measurements, is cited. Here we try to give a grounded visual picture of teleportation, indicating what material carrier transfers information of one or another type. We use a special version of the algebraic approach to quantum theory [2, 3].

The central notion used in this approach is an “observable”. An observable is an attribute of a physical system whose numerical value can be obtained using a certain measuring procedure. All the observables are assumed to be dimensionless. All the measurements are divided into reproducible and nonreproducible ones and also into compatible and incompatible ones. Compatible measurements are conducted using compatible measuring devices. If there exist compatible measuring devices for a group of observables, then such observables are said to be compatible (simultaneously measurable).

Postulate 1. Observables $\widehat{A}$ of a physical system are Hermitian elements of some C^*-algebra $\mathfrak{A}$ ($\widehat{A} \in \mathfrak{A}$, $\widehat{A}^* = \widehat{A}$) [4].

By $\mathfrak{A}_+$ ($\mathfrak{A}_+ \subset \mathfrak{A}$) we denote a set of observables.

Postulate 2. The set of compatible observables is a maximal real associative commutative subalgebra $\mathfrak{Q}_\xi$ of the algebra $\mathfrak{A}$ ($\mathfrak{Q}_\xi \subset \mathfrak{A}_+$).

The index ξ ranging a set Ξ distinguishes one such subalgebra from another.

We regard the set $\mathfrak{A}_+$ as a mathematical representation of the physical system under study and the sets $\mathfrak{Q}_\xi$ as mathematical representations of the corresponding classical subsystems of the physical system. These classical subsystems are open (interacting between themselves) and do not have their own dynamics. The state of a classical system is its attribute that uniquely predetermines the results of measurements of all the observables of the system. Therefore, we formulate the following postulate.

Postulate 3. The state of a classical subsystem whose observables are elements of the subalgebra $\mathfrak{Q}_\xi$ is described by a character of this subalgebra.

We recall that a homomorphic map of the associative commutative algebra to the set of numbers is called the character $\varphi_\xi(\cdot)$ of this algebra: $\widehat{A} \xrightarrow{\varphi_\xi} \varphi_\xi(\widehat{A})$, $\widehat{A} \in \mathfrak{Q}_\xi$ (see, e.g., [4]).

Because the observables belonging to the subalgebra $\mathfrak{Q}_\xi$ are compatible, there exists a set of measuring devices designed for compatible measurements of these observables. We say that these devices belong to the ξ-type.

The set $\mathfrak{A}_+$ of observables of a quantum system can be regarded as a collection of subsets $\mathfrak{Q}_\xi$. Therefore, the quantum system can be regarded as a set of corresponding open classical subsystems. Each observable of the quantum system belongs to a certain subset $\mathfrak{Q}_\xi$. Accordingly, if the states of all classical subsystems were known, then we could predict the result of measuring any observable of the quantum system. Based on this, we call the set $\varphi = [\varphi_\xi]$ ($\xi \in \Xi$) of functionals $\varphi_\xi(\cdot)$ each of which is the character of the corresponding subalgebra $\mathfrak{Q}_\xi$ the elementary state of a physical system. The following postulate is central in the described approach.

Postulate 4. The result of each individual measurement of the observables of a physical system is determined by the elementary state of this system.

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The elementary state is an attribute of a physical system and it is a local reality. It is impossible to determine the elementary state of a system uniquely in experiments. Only compatible measuring devices can be used to fix it. Using such devices, we can determine the functional $\varphi_\xi(\cdot)$ only for one value of ξ ($\xi = \eta$). All the other functionals $\varphi_\xi(\cdot)$ contained in the elementary state $[\varphi_\xi]$ remain undetermined. Figuratively speaking, we can say that the elementary state is a holographic image of a physical system. Using classical measuring devices, we can find only a plane image. In this case, each measurement changes an original holographic pattern. Therefore, it is impossible to obtain a complete holographic image.

We unite all the elementary states $[\varphi_\xi]$ having the same restriction to the subalgebra $\mathfrak{Q}_\eta$, i.e., the same functional φ_η, into the equivalence class $\{\varphi\}_\eta$. In experiments, thus, it is possible to uniquely fix only the equivalence class to which the elementary state of interest belongs. If we know that some elementary state $\varphi = [\varphi_\xi]$ belongs to the equivalence class $\{\varphi\}_\eta$, then we can uniquely predict what result can be obtained in the measurement of the observable $\widehat{A} \in \mathfrak{Q}_\eta$: this result is $\varphi_\eta(\widehat{A})$. But if $\widehat{A} \notin \mathfrak{Q}_\eta$, then it is impossible to say anything definite about the measurement result. For different elementary states belonging to $\{\varphi\}_\eta$, the measurement results are different. The quantum state fixed by certain values of the observable $\widehat{A}$ from the subalgebra has such physical properties in the standard quantum mechanics $\mathfrak{Q}_\eta$.

If $\{\varphi\}_\eta$ is endowed with the structure of a probability space, then we can use standard probability theory methods (see, e.g., [5, 6]) to easily construct the functional $\Psi_\eta\big(\widehat{A}\big)$ that specifies the mean of an observable $\widehat{A}$ in the equivalence class $\{\varphi\}_\eta$.

Postulate 5. The probability structure of the equivalence class $\{\varphi\}_\eta$ is such that the functional $\Psi_\eta\big(\widehat{A}\big)$ is linear on the algebra $\mathfrak{A}$.

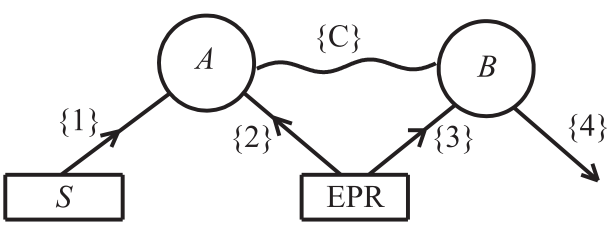
Having the C^*-algebra $\mathfrak{A}$ and the linear functional $\Psi(\cdot)$ on this algebra and using the canonical Gelfand–Naimark–Segal construction (see, e.g., [7]), we can realize the representation of this algebra by bounded linear operators in a Hilbert space $\mathfrak{H}$: $\widehat{A} \leftrightarrow \Pi(\widehat{A})$, $\widehat{A} \in \mathfrak{A}$, $\Pi(\widehat{A}) \in \mathfrak{B}(\mathfrak{H})$, where $\mathfrak{B}(\mathfrak{H})$ is the set of bounded linear operators in $\mathfrak{H}$. In this case, the mean $\langle \widehat{A} \rangle$ of the observable $\widehat{A}$ in the quantum state Ψ can be expressed as the mathematical expectation of the operator $\Pi(\widehat{A})$: $\langle \widehat{A} \rangle = \Big\langle \Psi \Big| \Pi(\widehat{A}) \Big| \Psi \Big\rangle$, where $|\Psi\rangle \in \mathfrak{H}$ is the corresponding vector of the Hilbert space.

The so-called entangled states play the central role in the quantum teleportation procedure. In the literature, the entangled states typical of a two-particle system in which each of the particles can be in two orthogonal quantum states $|\pm\rangle$ are most often considered:

$$\begin{aligned} \big|\Psi^{(-)}\big\rangle_{12} &= \frac{1}{\sqrt{2}}\big[\,|+\rangle_1|-\rangle_2 - |-\rangle_1|+\rangle_2\big], & \big|\Psi^{(+)}\big\rangle_{12} &= \frac{1}{\sqrt{2}}\big[\,|+\rangle_1|-\rangle_2 + |-\rangle_1|+\rangle_2\big], \\ \big|\Phi^{(-)}\big\rangle_{12} &= \frac{1}{\sqrt{2}}\big[\,|+\rangle_1|+\rangle_2 - |-\rangle_1|-\rangle_2\big], & \big|\Phi^{(+)}\big\rangle_{12} &= \frac{1}{\sqrt{2}}\big[\,|+\rangle_1|+\rangle_2 + |-\rangle_1|-\rangle_2\big]. \end{aligned} \tag{1}$$

These states are often called the Bell states. The state $\big|\Psi^{(-)}\big\rangle_{12}$ is usually considered in the discussion of the Einstein–Podolsky–Rosen paradox [8] and is therefore often called the EPR state. The system consisting of two particles with spin 1/2 was considered in the version proposed by Bohm [9]. Then, $|+\rangle$ is the quantum state with the spin projection on the z-axis equal to $+1/2$, and $|-\rangle$ is the state with the projection equal to $-1/2$.

In the state $\big|\Psi^{(-)}\big\rangle_{12}$, the total spin $\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$ is zero. The characteristic feature of the state $\big|\Psi^{(-)}\big\rangle_{12}$ is its spherical symmetry. Therefore, it preserves its form if the projections on an arbitrary direction $\boldsymbol{n}$ are considered instead of the projections on the z-axis. In this state, for example, the relation $S_{\boldsymbol{n}1} + S_{\boldsymbol{n}2} = 0$ holds, where $S_{\boldsymbol{n}1}(S_{\boldsymbol{n}2})$ is the spin projection of the i-th particle on the direction $\boldsymbol{n}$.



The elementary state of one particle of the EPR pair is the negative copy of the elementary state of the other particle. Therefore, a measurement of the value of an observable of the first particle is automatically a measurement of the corresponding observable of the second particle irrespective of the location of this particle. Such a measurement is said to be indirect.

Figure shows a scheme of quantum teleportation.

Here, S is the source of the initial state, EPR is the source of EPR pairs, A is the analyzer of the Bell states (Alice), B is the unitary converter (Bob), $\{C\}$ is the classical communication channel, $\{1\}$ is the carrier of the initial teleported state, $\{2\}$–$\{3\}$ is the EPR pair, and $\{4\}$ is the carrier of the final teleported state.

We give the standard description of the teleportation scheme (see, e.g., [10]). Each of the particles $\{1\}$,

$\{2\}$, $\{3\}$, and $\{4\}$ participating in the process can be in one of the quantum levels $|+\rangle$ or $|-\rangle$. The source S emits particle $\{1\}$ in the quantum state $|\Psi\rangle_1 = \alpha|+\rangle + \beta|-\rangle$, where $|\alpha|^2 + |\beta|^2 = 1$. In the general case, α and β can be unknown. The EPR source emits particles $\{2\}$ and $\{3\}$ in the quantum state $|\Psi^{(-)}\rangle_{23}$ (see formula (1)). The quantum state of the three-particle system consisting of particles $\{1\}$, $\{2\}$, and $\{3\}$ is described by the vector $|\Psi\rangle_{123} = |\Psi\rangle_1 \bigotimes |\Psi^{(-)}\rangle_{23}$ which can be decomposed in terms of the Bell states of particles $\{1\}$ and $\{2\}$:

$$\begin{aligned} |\Psi\rangle_{123} = \frac{1}{2}\Big\{ |\Psi^{(-)}\rangle_{12}(-\alpha|+\rangle_3 - \beta|-\rangle_3) + |\Psi^{(+)}\rangle_{12}(-\alpha|+\rangle_3 + \beta|-\rangle_3) + \\ + |\Phi^{(-)}\rangle_{12}(\alpha|-\rangle_3 + \beta|+\rangle_3) + |\Phi^{(+)}\rangle_{12}(\alpha|-\rangle_3 - \beta|+\rangle_3)\Big\}. \end{aligned} \tag{2}$$

Using the analyzer A, Alice measures to find the Bell states of the four ones possible for the accessible particles $\{1\}$ and $\{2\}$. For example, we suppose that she obtains $|\Psi^{(-)}\rangle_{12}$ as a result. Then the three-particle system collapses to the state $|\Psi'\rangle_{123} = |\Psi^{(-)}\rangle_{12}(-\alpha|+\rangle_3 - \beta|-\rangle_3)$ after such a measurement according to the projection postulate. Alice broadcasts her discovery that the particles $\{1\}$ and $\{2\}$ are in the state $|\Psi^{(-)}\rangle_{12}$ over the classical communication channel. Bob, doing nothing, transmits particle $\{3\}$. This particle is now in the state $|\Psi\rangle_4 = (-\alpha|+\rangle_3 - \beta|-\rangle_3)$, which coincides with the state $|\Psi\rangle_1$. It is very difficult to imagine how Alice, not acting physically on particle $\{3\}$, could make it pass to the quantum state of particle $\{1\}$. In this case, Alice even knew nothing about that state.

We now discuss how the same teleportation process can be described using the notion of an elementary state [11]. In this case, a whole beam of particles $\{1\}$ that are in the different elementary states (but all belonging to the same equivalence class) corresponds to the quantum state $|\Psi\rangle_1$. Accordingly, a beam of EPR pairs $\{2\}$–$\{3\}$ rather than one pair is required in the experiments. In the quantum state $|\Psi\rangle_1$, the numbers α and β specify a direction $\boldsymbol{n}$ along which the spin projection of each particle $\{1\}$ of the beam is definitely equal to 1/2. Let the z-axis be along the direction $\boldsymbol{n}$. Then, for the spin projection, the equality $S_z = +1/2$ holds in the quantum state $-\alpha|+\rangle_3 - \beta|-\rangle_3$, the equality $S_z = -1/2$ holds in the state $-\alpha|+\rangle_3 + \beta|-\rangle_3$, the equality $S_x = +1/2$ holds in the state $\alpha|-\rangle_3 + \beta|+\rangle_3$, and the equality $S_x = -1/2$ holds in the state $|\alpha|-\rangle_3 - \beta|+\rangle_3$.

We now regard the analyzer A in combination with particle $\{1\}$ as a measuring device. The action of this combined measuring device on the beam of particles $\{2\}$ can be interpreted two ways (see formula (2)). On one hand, this device divides the beam of particles $\{2\}$ into four subbeams in each of which particles $\{2\}$ (in combination with particles $\{1\}$) are in one of the Bell states. This result is fixed by the analyzer A. On the other hand, the particles $\{2\}$ in each of these four subbeams have definite values of the spin projections either on the z-axis or on the x-axis. Because of the strict correlation between the elementary states of particles $\{2\}$ and $\{3\}$, the beam of particles $\{3\}$ automatically divides into four subbeams in each of which particles $\{3\}$ have certain spin projections. That is, we have a typical example of an indirect measurement of the spin projection for particle $\{3\}$ in this case. Using the classical communication channel, Alice reports the result of such an indirect measurement to Bob. Bob applies the corresponding unitary transformation to particles $\{3\}$. As a result of this measurement, only some information about this elementary state needed for Bob's subsequent actions is obtained.

We call attention to the fact that the elementary state of particle $\{3\}$ does not become the same as that of particle $\{1\}$ after all the described manipulations. These particles only turn out to be in the same equivalence class. Thus, particle $\{3\}$ does not become an exact copy of particle $\{1\}$; therefore, the term "teleportation" used to describe this procedure is not especially appropriate.

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