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СПЕЦИАЛЬНОЕ СТОХАСТИЧЕСКОЕ ПРЕДСТАВЛЕНИЕ КВАНТОВОЙ МЕХАНИКИ И СОЛИТОНЫ

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Предложено специальное стохастическое представление квантовой механики. В основу этого представления положена линейная комбинация солитонных решений некоторых нелинейных уравнений поля, причем частицы идентифицируются с солитонными конфигурациями. Оказывается, что волновая функция частицы — это вектор в случайном гильбертовом пространстве. Многочастичная волновая функция построена с помощью многосолитонных конфигураций. Показано, что в точечном пределе, когда собственный размер солитонной конфигурации исчезает, восстанавливаются основные принципы квантовой механики. В частности, средние значения физических наблюдаемых получаются как эрмитовы формы, порожденные самосопряженными операторами, а связь спина со статистикой получается как следствие протяженного характера частиц-солитонов. Статья рекомендована к печати программным комитетом международной научной конференции "Математическое моделирование и вычислительная физика 2009" (MMCP2009, http://mmcp2009.jinr.ru).

Ключевые слова: стохастическое представление, солитоны, случайное гильбертово пространство.

1. Introduction. One could imagine two possible ways of constructing quantum theory of extended particles. The first one was suggested by L. de Broglie [1] and A. Einstein [2] and consists in using soliton solutions to some nonlinear field equations for the description of the particles' internal structure. The second way supposes the introduction of minimal elementary length ℓ_0 [3] and/or nonlocal interaction [4]. The aim of our work is to show the consistency of the first approach with the principles of quantum mechanics (QM). To this end, we introduce a special stochastic representation of the wave function using solitons as images of the extended particles. To realize this approach, suppose that a field ϕ describes n particles-solitons and has the form $\phi(t,\boldsymbol{r}) = \sum_{k=1}^{n} \phi^{(k)}(t,\boldsymbol{r})$, and the same for the conjugate momenta $\pi(t,\boldsymbol{r}) = \dfrac{\partial \mathcal{L}}{\partial \phi_t} = \sum_{k=1}^{n} \pi^{(k)}(t,\boldsymbol{r})$, $\phi_t = \dfrac{\partial\phi}{\partial t}$. Let us define the auxiliary functions

$$\varphi^{(k)}(t,\boldsymbol{r}) = \frac{1}{\sqrt{2}}\left(\nu_k \phi^{(k)} + \frac{\boldsymbol{i}\pi^{(k)}}{\nu_k}\right) \qquad (1)$$

with the constants ν_k satisfying the normalization condition $\hbar = \int d^3x \left|\varphi^{(k)}\right|^2$. Now we define the analog of the wave function in the configurational space $\mathbb{R}^{3n} \ni \boldsymbol{x} = \{\boldsymbol{r}_1, \ldots, \boldsymbol{r}_n\}$ as follows:

$$\Psi_N(t,\boldsymbol{r}_1,\ldots,\boldsymbol{r}_n) = (\hbar^n N)^{-1/2} \sum_{j=1}^{N} \prod_{k=1}^{n} \varphi_j^{(k)}(t,\boldsymbol{r}_k). \qquad (2)$$

Here $N \gg 1$ stands for the number of trials (observations) and $\varphi_j^{(k)}$ is the one-particle function (1) for the j-th trial. Earlier [5, 6] it was shown that the quantity $\rho_N = \dfrac{1}{(\triangle\vee)^n} \displaystyle\int_{(\triangle\vee)^n \subset \mathbb{R}^{3n}} d^{3n}x \left|\Psi_N\right|^2$, where $\triangle\vee$ is the elementary volume supposed to be much greater than the proper volume of the particle ${\ell_0}^3 = \vee_0 \ll \triangle\vee$, plays the role of probability density for the distribution of solitons' centers.

It is interesting to emphasize that the solitonian scheme in question contains also the well-known spin-statistics correlation [7]. Namely, if $\varphi_j^{(k)}$ is transformed under the rotation by the irreducible representation $D^{(J)}$ of $SO(3)$ with the weight J, then the transposition of two identical extended particles is equivalent to the relative 2π-rotation of $\varphi_j^{(k)}$ that gives the multiplication factor $(-1)^{2J}$ in Ψ_N. To show this property, suppose

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that our particles are identical, i.e., their profiles $\varphi_j^{(k)}$ may differ in phases only. Therefore, the transposition of the particles with the centers at $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ means the π-rotation of 2-particle configuration around the median axis of the central vector line $\boldsymbol{r}_1 - \boldsymbol{r}_2$. Due to extended character of the particles, however, to restore the initial configuration, one should perform additional proper π-rotations of the particles. The latter operation being equivalent to the relative 2π-rotation of particles, it results in the aforementioned multiplication of Ψ_N by $(-1)^{2J}$. Under the natural supposition that the weight J is related with the spin of particles-solitons, one infers that the many-particles wave function (2) should be symmetric under the transposition of the two identical particles if the spin is integer but should be antisymmetric if the spin is half-integer (the Pauli principle).

Now let us consider the measuring procedure for some observable A corresponding, due to Noether's theorem, to the symmetry group generator $\widehat{M}_A$. For example, the momentum $\boldsymbol{P}$ is related with the generator of space translation $\widehat{M}_{\boldsymbol{P}} = -\boldsymbol{i}\,\nabla$, the angular momentum $\boldsymbol{L}$ is related with the generator of space rotation $\widehat{M}_{\boldsymbol{L}} = \boldsymbol{J}$ and so on. As a result one can represent the classical observable A_j for the j-th trial in the form $A_j = \int d^3x\, \pi_j \boldsymbol{i} \widehat{M}_A \phi_j = \int d^3x\, \varphi_j^* \widehat{M}_A \varphi_j$. The corresponding mean value is

$$\mathbb{E}(A) \equiv \frac{1}{N}\sum_{j=1}^{N} A_j = \frac{1}{N}\sum_{j=1}^{N}\int d^3x\, \varphi_j^* \widehat{M}_A \varphi_j = \int d^3x\, \Psi_N^* \widehat{A} \Psi_N + O\left(\frac{\mathsf{V}_0}{\triangle \mathsf{V}}\right), \tag{3}$$

where the Hermitian operator $\widehat{A}$ reads $\widehat{A} = \hbar \widehat{M}_A$. Up to the terms of the order $\dfrac{\mathsf{V}_0}{\triangle\mathsf{V}} \ll 1$, we obtain the standard QM rule (3) for the calculation of mean values. Thus, we conclude that in the solitonian scheme the spin-statistics correlation stems from the extended character of particles-solitons. With the particles in QM being considered as point-like ones, however, it appears inevitable to include the transpositional symmetry of the wave function as the first principle (cf. Hartree–Fock receipt for fermions). Various aspects of the fulfillment of the QM correspondence principle for the Einstein–de Broglie's soliton model were discussed in [5–11]. The fundamental role of the gravitational field in the de Broglie–Einstein solitonian scheme was discussed in [7]. The soliton model of the hydrogen atom was developed in [8, 9]. The wave properties of solitons were described in [10]. The dynamics of solitons in external fields was discussed in [11].

As a result, we obtain the stochastic realization (2) of the wave function Ψ_N which can be considered as an element of the random Hilbert space $\mathcal{H}_{\text{rand}}$ with the scalar product $(\psi_1, \psi_2) = \mathbb{M}(\psi_1^* \psi_2)$ with $\mathbb{M}$ standing for the expectation value. As a crude simplification, one can assume that the averaging in this product is taken over the random characteristics of particles-solitons, such as their positions, velocities, phases, and so on. It is important to recall once more that the correspondence with the standard QM is retained only in the point-particle limit $(\triangle\mathsf{V} \gg \mathsf{V}_0)$ for $N \to \infty$.

In conclusion we emphasize that, using special stochastic representation of the wave function, we show that the main principles of quantum theory can be restored in the limit of the point-like particles. Moreover, we prove that the spin-statistics correspondence (the Pauli principle) can be considered as the natural consequence of the internal structure of elementary particles.

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